Thus, the rarefaction in the scattering regime as indicated by r_p^1 (or r_p) scales with the same rarefaction parameter as the shock broadening in the transition regime. Evidently the parameter $D(P_sP_{B_\infty})^{1/2}/T$ can be used to scale certain aspects of plume rarefaction for a wide range of conditions.

4. Conclusions

The transition from continuum flow to perfect vacuum expansion of the plume from an underexpanded sonic orifice has been described. Certain features of the entire rarefaction process can be scaled with the rarefaction parameter $D(P_sP_{B_\infty})^{1/2}/T$ that has been derived in this paper. The broadening of the shock waves bounding the plume, the penetration of background gas into the plume and the eventual transition to perfect vacuum expansion have all been considered.

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Drag Coefficients for Free Molecule Flow in the Velocity Range 7–37 km/sec

J. W. Boring* and R. R. Humphris†
University of Virginia, Charlottesville, Va.

Measurements have been made of the momentum transfer by a beam of N_2 molecules to solid surfaces for molecular energies of 8–200 ev (velocity range 7–37 km/sec). The results of these measurements allow one to calculate drag coefficients for the situation where nearly monoenergetic molecules all moving in the same direction impinge upon a solid convex body. Drag coefficients for N_2 molecules striking spheres of Echo I and Echo II satellite material are found to be in the range 1.9–2.2.

Introduction

SINCE the advent of Earth satellites there has been intense interest in particle-surface interactions for relative velocities in the range 7–11 km/sec. However, there is only a small amount of experimental information available concerning these interactions for neutral species because of the inherently awkward techniques that must be employed in order to achieve velocities in this range. In using satellites to gain information concerning the density of the earth's upper atmosphere, one is particularly interested in the drag produced on the satellite by the atmosphere and hence in the manner in which atoms and molecules exchange momentum with the surfaces of the satellite. The present paper describes laboratory measurements of the momentum transfer to solid surfaces by N_2 molecules in the energy range 8–200 ev (velocity range 7–37 km/sec). The results are obtained as a function of the angle of incidence, thereby permitting one to calculate drag coefficients for solid bodies of arbitrary convex shape moving through a gas that has thermal motion that is negligible compared to the velocity of the body.

Consider now the drag on a satellite moving through a rarefied atmosphere (free molecular flow) with a speed large, compared to the thermal motion of the atmospheric molecules. The drag coefficient can then be expressed as

$$C_D = \frac{F}{\frac{1}{2}A\rho v_0^2} = 2\left[1 + \frac{1}{A}\int_s \frac{P_m}{P_0}\cos\theta da\right] \quad (1)$$

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* Associate Professor, Department of Aerospace Engineering and Engineering Physics.

† Senior Scientist, Research Laboratories for the Engineering Sciences.

where F is the drag force, A is the cross sectional area of the satellite projected on the plane normal to the direction of motion, ρ is the atmospheric density, v_0 is the relative velocity of the satellite through the atmosphere, P_0 is the corresponding molecular momentum, P_m is the average component of momentum of reflected molecules along the direction of motion (taken positive when opposite to P_0), θ is the angle of incidence of molecules (measured from the normal to the surface) striking an element of surface da, and the integral extends over the front surface of the satellite. The above expression may therefore be used to calculate the drag coefficient for a body of convex shape (so that double reflections are not possible) moving through a one-component atmosphere if the ratio P_m/P_0 is known as a function of θ for a given v_0. If the atmosphere contains several components, then it is necessary to know P_m/P_0 for each molecular species as well as the proportion of each present. To take a simple example, if one considers a flat plate moving so that its surface is normal to the direction of motion, then $\theta = 0$ for the entire surface and we get

$$C_D = 2(1 + P_m/P_0) \text{ (flat plate)}$$

If the momentum of the reflected molecules is small compared to the incident momentum ($P_m/P_0 \ll 1$) then $C_D \simeq 2$, whereas if the molecules are reflected back along the direction of v_0 (specularly) with a speed equal to v_0, then $P_m = P_0$ and $C_D = 4$. One therefore, would expect the measured value of C_D for a flat plate to be somewhere between these limits: $2 < C_D < 4$. For a convex body it is conceivable that for a considerable fraction of the surface P_m is in the same direction as P_0 and hence is negative, leading to the possibility of values of C_D less than 2.

The present paper is concerned with the measurement in the laboratory of P_m/P_0 as a function of v_0 and θ for N_2 molecules incident on several surfaces, and the calculation of drag coefficients from the results of these measurements.

Experimental Method

The general procedure in measuring P_m/P_0 is to produce a beam of molecules having a known energy corresponding to satellite velocities (the energy in the case of N_2 molecules is 8–18 ev), allow the molecules to strike a test surface at a chosen angle of incidence, and measure the component of force on the test surface along the beam direction. If the rate at which the molecules strike the surface (molecules/sec) is determined, then the force divided by the rate gives ($P_0 + P_m$), and since P_0 is already known from a knowledge of the energy and mass of the beam molecules, then one has sufficient information to determine P_m/P_0.

The neutral beam is formed by creating a beam of N_2^+ ions of the desired energy and allowing a portion to become neutralized by charge transfer in N_2 gas, with the fast N_2 molecules thus formed having essentially the same velocity as the ions. The test surface is mounted on a sensitive torsion balance for measurement of the force produced by the beam, with the balance also being used to measure the beam flux through the use of a "momentum trap."

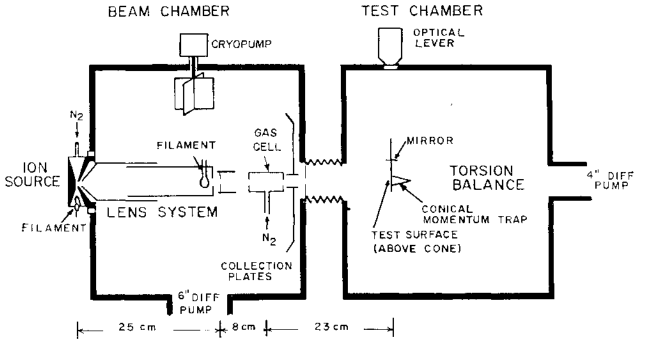

Fig. 1 Schematic diagram of apparatus.

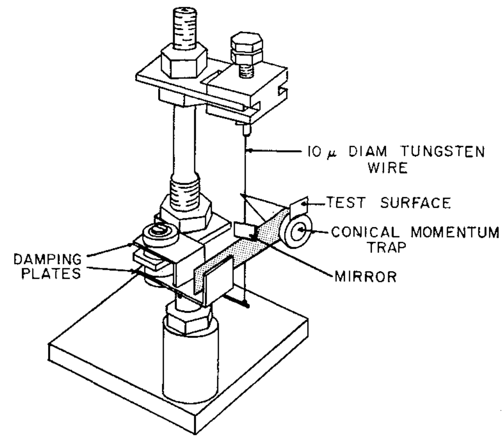

Fig. 2 Torsion balance.

A schematic drawing of the entire experimental system is shown in Fig. 1. The apparatus is mounted in two separate vacuum chambers, a beam chamber and a test chamber. The beam chamber contains the ion source, electrostatic lens system, and neutralization cell. The test chamber contains the test surface, which is mounted on a torsion balance used to measure the force produced by the beam on the surface. The test chamber is placed on a large concrete pier which is isolated from the laboratory floor to reduce mechanical vibrations in the torsion balance. The two chambers are connected by a metal bellows which allows movement of the beam chamber so that the beam can be moved with respect to the test surface.

The torsion balance which is used to measure the force produced by the beam on the test surface was patterned after one described by Pearson and Wadsworth[1] and uses electrostatic damping and an optical lever[2] for measuring the angular deflection of the balance arm. This balance is relatively rugged but is capable of detecting forces as small as 2×10^{-8} dyne. The balance configuration used in our work is shown in Fig. 2. The torsion fiber is 10 μ tungsten, and both damping plates are mounted on the same end of the balance arm. The test surface is mounted on the other end of the balance arm along with a momentum trap for measuring the beam flux. The procedure is to allow the beam to enter the momentum trap, measure the balance deflection, and calculate the beam flux under the assumption that the molecules leaving the trap have a Maxwellian velocity distribution which is characteristic of the temperature of the trap. (The results are not very sensitive to the precise validity of this assumption since the average momentum of nearly thermally accommodated molecules leaving the trap is much less than that of the incident molecules.) The beam is then moved upward mechanically so that it strikes the test surface and the balance deflection is again observed, giving a measurement of the force produced by the beam. This method of measuring the beam flux has the advantage that the absolute calibration of the balance is not needed, since two balance deflections are being compared in order to find the average momentum transferred to the test surface by the molecules of the beam. In each measurement the ion beam is turned on and off and the resulting balance deflection is taken as a measure of the force of interest. The effect of all forces on the balance other than that produced by the beam (such as that caused by the neutralizing gas) are thus eliminated. The momentum trap is 4.1 cm long with an apex angle of 22° and with a 0.9-cm-diam hole. The ratio of the hole area to the total internal surface area of the cone is 0.052, so that one would expect entering molecules to experience, on the average, around 20 collisions with the surface of

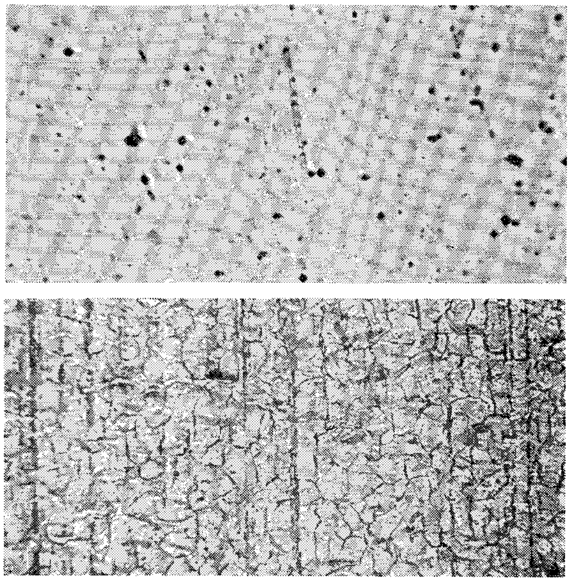

Fig. 3 Photomicrographs of surfaces: top—Echo I, bottom—Echo II; (400x) magnification.

the box before leaving. This means that if one assumes that the thermal accommodation coefficient for a single collision of a molecule with the inner surface of the cone is greater than around 0.2, the assumption that the molecules leaving the box have an average velocity that is characteristic of the box temperature is well satisfied. Although there have been no direct measurements of the accommodation coefficient in the ev energy range, the measurements at thermal energies[3] indicate that for gas covered surfaces the accommodation coefficient is generally greater than 0.5. Theoretical consideration of the particle-surface interaction as being hard-sphere at these energies also leads one to think that the accommodation coefficient for these ev studies (with or without adsorbed gases) will be in excess of 0.5.

In a measurement, the beam is allowed to strike the test surface and the balance deflection is recorded. The beam chamber is then moved downward mechanically so that the beam enters the momentum trap, and the corresponding balance deflection is recorded. The ratio of these two deflections then gives

$$R = (P_0 + P_m)/(P_0 + P_a)$$

where P_a is the momentum due to the molecules leaving the momentum trap and has a maximum value for these experiments of around 0.055 P_0. If one assumes that the molecules collide with the walls of the momentum trap a sufficiently large number of times, so that they are in thermal equilibrium with the walls and leave with a corresponding velocity and angular distribution, then by knowing the temperature of the walls, P_a can be calculated, and P_m/P_0 can be easily computed from $P_m/P_0 = R(1 + P_a/P_0) - 1$.

Generally, four or five measurements are taken with the beam striking the test surface, then a similar number with the beam entering the momentum trap, and then another set with the test surface. The average for the test surface is then compared to the average for the momentum trap. The fact that a complete measurement includes two sets for the test surface tends to minimize the effect of slowly changing beam conditions.

The measurements were performed for several different test surfaces. The entire question of surface condition in experiments such as these involves a number of uncertainties. In considering the application of the measurements to satellite studies of the density of the Earth's upper atmosphere, one would like to make the measurements using samples of actual satellite surfaces which have the same surface condition as that of the satellite in orbit (especially regarding adsorbed gases on the surface). There are two reasons why this desirability cannot be achieved at present. First, the condition of the satellite's surface is to a large extent unknown. It depends on the preparation of the satellite, its environment in orbit, and possible continual emission of gases from portions of the satellite. Second, even the most advanced laboratory techniques are not presently capable of specifying precisely the condition of a given surface. It is possible, however, that some aspects of the molecule-surface interaction are not especially sensitive to the exact nature of the surface, particularly aspects that involve averages over a number of parameters. Since the momentum transfer measurements described here provide a rather coarse study of the interaction, the following philosophy has been adopted with regard to surface condition.

The measurements are performed for several test surfaces, but the exact condition of the surface is not rigidly controlled. The surfaces are handled carefully before placing them in the vacuum system so as to prevent their being contaminated by oils, fingerprints, etc., but no attempt is made to remove adsorbed gases from the surfaces after they are in the vacuum system and the measurements are performed at pressures ($\sim 10^{-6}$ torr) such that a clean surface (no adsorbed gases) cannot be maintained. If the results of the measurements indicate that only the gross character of the surface (such as surface roughness) affects the momentum transfer, then one might conclude that the surfaces can be adequately characterized for this particular type of measurement. Measurements that investigate finer details of the interaction, such as the angular and velocity distribution of the reflected particles, may require considerably more accurate surface characterization.

Results

The objective in these measurements is to investigate P_m/P_0 as a function of molecule energy and angle of incidence for several test surfaces, including samples of surfaces used on actual satellites. The surfaces used in these studies were samples of material used in the Earth satellites Echo I and Echo II. Echo I is aluminum evaporated on Mylar and Echo II is aluminum with a coating of Alodine, an amorphous phosphate used to control the light absorption characteristics of the surface.[4] Photomicrographs of these two surfaces illustrating the gross roughness are shown in Fig. 3. The results of the measurements are presented in Figs. 4 and 5. One of the principal factors in determining the nature of the particle-surface interaction is the ratio of the masses of the incident molecule and the surface atoms that it strikes. For this reason a measurement was made for a gold surface (mass number 197) at $\theta = 0°$ to see if the results are affected by a large change in the mass number of the base material. The fact that the results for gold were essentially the same as for the

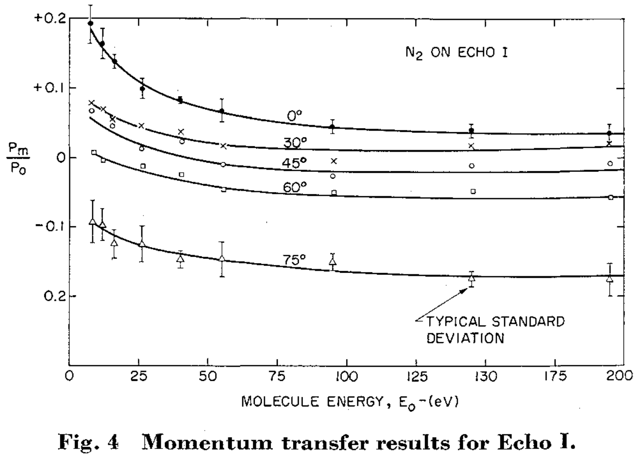

Fig. 4 Momentum transfer results for Echo I.

other surfaces indicates that under the conditions of these measurements the interaction with adsorbed gases appears to predominate.

A straightforward investigation of the dependence of the ratio P_m/P_0 on the angle of incidence would involve mounting the test surfaces on the balance vane so that the beam molecules strike the surface at the chosen angle, but with the beam direction still perpendicular to balance vane. This means that the balance must be modified or reconstructed for each new angle. At angles of incidence less than 30° this procedure was followed with success. At larger angles, however, it was found that the combination of the mass of the momentum trap and that of the larger test surface needed to intercept all of the beam passing through the collimating aperture, caused the balance to be intolerably noisy. Since all the surfaces studied gave essentially identical results at $\theta = 0°$, it was decided that the measurements at larger angles would be performed by eliminating the momentum trap and using one of the surfaces studied (at $\theta = 0°$) as a reference surface, thereby reducing the mass mounted on the balance arm. The ratio P_m/P_0 for the larger angles can thus be obtained by comparing deflections for the inclined surface and the reference surface, and then using the results obtained previously that provided a comparison of the reference surface with the momentum trap. If the deflection for the inclined surface divided by the deflection for the reference surface is called S, then

$$S = (P_0 + P_m)/(P_0 + P_m{}^0) = (1 + P_m/P_0)/(1 + P_m{}^0/P_0)$$

where $P_m{}^0$ is the value of P_m for the reference surface for $\theta = 0$. Also,

$$R^0 = (P_0 + P_m{}^0)/(P_0 + P_a) = (1 + P_m{}^0/P_0)/(1 + P_a/P_0)$$

where R^0 is the value of R for the reference surface at $\theta = 0$. Eliminating $P_m{}^0/P_0$ between the expressions for S and R^0 and solving for P_m/P_0, one gets

$$P_m/P_0 = SR^0(1 + P_a/P_0) - 1$$

This is the expression used for obtaining P_m/P_0 for the larger angles of incidence where one is comparing the force on an inclined surface to that on a reference surface.

In Fig. 4 for the Echo I surface, error bars are shown for the measurements at $\theta = 0°$ and $\theta = 75°$, and are representative of the corresponding uncertainties in the other measurements. The error bars give the standard deviation in the mean value of P_m/P_0 as calculated from a number of measurements at a given energy, and therefore represent the result of random fluctuations from the mean value caused by system noise, etc. No inclusion has been made of possible systematic errors in the measurements, but it is felt that these should be small since the measurements involve a comparison of two determinations of the same type of quantity (balance deflection) that were repeated a number of times for each surface and angle over a period of several months. The measurements were made for at least two samples of each surface material, with the results indicating no difference between samples.

One can compare the present results with other experiments only on a semiquantitative basis since the available results correspond to work that is significantly different in type of incident particle and energy range. The work of Mair, Viney, and Colligan[5] for N_2^+ ions incident normally on a copper surface indicates that P_m/P_0 varies from about 0.6 at 7 ev to 0.3 at 100 ev. These values are significantly higher than the corresponding ones for the present work with neutral N_2 molecules. Knechtel and Pitts[6] have studied momentum accommodation of Ar^+ ions on several surfaces as a function of energy and incident angle. Their measurements were made under conditions where one expects the surface to be covered with adsorbed gases, and they found no significant difference in their results for gold and aluminum surfaces, in agreement with the results of the present paper. They found for 15 ev ions values of P_m/P_0 around 0.1 for $\theta = 0°$ and values as low

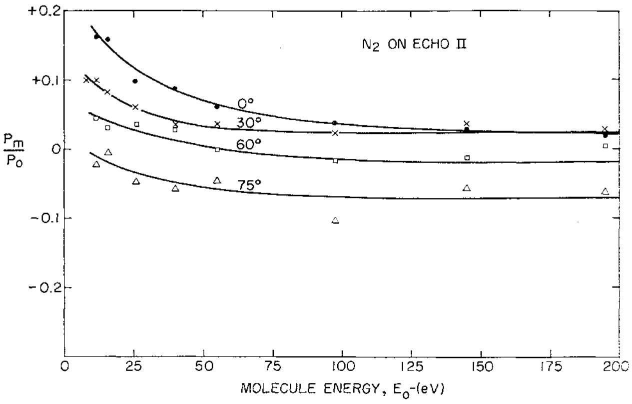

Fig. 5 Momentum transfer results for Echo II.

as -0.2 for $\theta = 60°$, in agreement with the general trend of our results. However, they obtained positive values of P_m/P_0 at 50 ev for all angles up to 60°, whereas our results yield negative values for the larger angles at all energies up to 200 ev. Abauf and Marsden[7] measured the momentum accommodation of neutral Ar atoms on several surfaces at energies up to 4 ev. Their results yield a value of P_m/P_0 for $\theta = 0°$ of around 0.4, which appears reasonable when compared to our results for N_2 at higher energies.

In these comparisons use has been made of the following expression:

$$P_m/P_0 = (2 - \sigma - \sigma') \cos^2\theta - (1 - \sigma) + (P_a/P_0) \cos\theta$$

which relates P_m/P_0 to the normal (σ') and tangential (σ) momentum accommodation coefficients.

Comparison with existing theories on particle-surface interactions can be made only on a qualitative basis, largely because in the present experiment the surface state is not sufficiently well characterized for meaningful quantitative comparisons. The work of Goodman[8] indicates that if one considers the interaction at high energies as being between hard spheres one expects P_m/P_0 to decrease with increasing energy and asymptotically approach a constant value. This is the behavior seen in the present experimental results. Similar considerations[9] lead to the conclusion that at a given energy the dependence on angle should be generally that seen in the experimental data: a positive value of P_m/P_0 at $\theta = 0°$ and a decrease to negative values at the larger angles.

Drag Coefficients

One of the purposes of these measurements has been to allow one to calculate drag coefficients for bodies moving in free-molecular flow with speeds in the satellite range. With the experimental results of the last section and Eq. (1), one can calculate drag coefficients for a body of any convex shape. In this section such calculations for spherical bodies are performed.

In order to do the integration indicated in Eq. (1) over a sphere, it is necessary to know P_m/P_0 as a function of θ. Figure 6 shows P_m/P_0 vs $\cos\theta$ for two energies. For a sphere Eq. (1) can be written as

$$C_D = 2\left(1 + 2\int_0^{\pi/2} \frac{P_m}{P_0} \sin\theta \cos\theta d\theta\right) \qquad (2)$$

Taking the data points of Fig. 6 and fitting them with straight line segments (three or less) in the range $0.26 < \cos\theta < 1.00$ one can easily calculate the contribution to the drag coefficient for parts of the sphere where θ is less than 75°. Since no data was taken for angles greater than 75°, then some extrapolation to larger angles must be used. Consider two such extrapolation procedures as bounds on the actual behavior of the curve

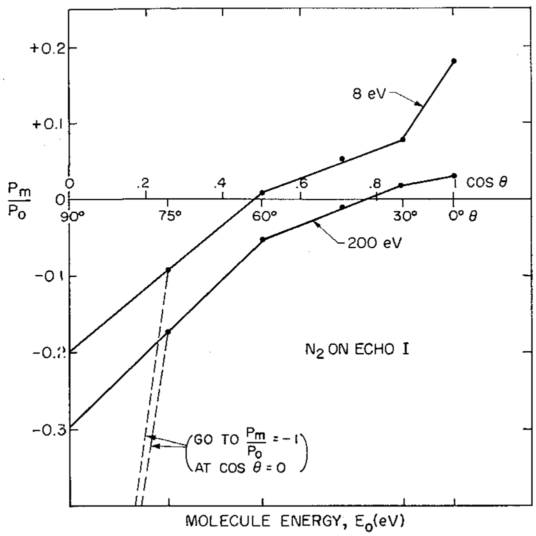

Fig. 6 Typical results as a function of cosθ.

in this region. 1) An extension of the straight line segment used for angles slightly less than 75° to angles 75° < θ < 90° will give the upper bound to the actual curve, and 2) a straight line drawn from the data point at 75° to $P_m/P_0 = -1$ at $\theta = 90°$ will give the lower bound. A plot of the drag coefficients obtained by these two methods is shown in Fig. 7. It is seen that these two extremes amount to about a ±1% difference in drag coefficient.

There appears to be a uniform difference between the curves for the two surfaces of about 3%, with Echo II indicating the higher drag coefficient. This higher value is probably due to the difference in surface roughness of the two materials. As noted in Fig. 3, the Echo II surface has many random cracks or deep valleys, on a macrosopic scale, as compared to the Echo I surface. When the test surface angle is greater than 0°, roughness would tend to increase the reflected component P_m. Thus, the ratio P_m/P_0 is increased, resulting in a higher value for the drag coefficient C_D.

The error bars on the low energy data (shown typically in Fig. 4) would lead to about a ±1% uncertainty in the drag coefficient, which along with the uncertainty in the large angle extrapolation procedure would lead to a total uncertainty in the drag coefficients of about 2% for the lowest energies of the curves of Fig. 7.

The drag coefficients for the surface studied are seen to be slightly greater than 2 at low energies, a result one would also get by assuming that the reflected molecules are thermally accommodated to the surface. It should be pointed out, then, that the $C_D \simeq 2$ for spheres obtained from the present experimental results comes about because the appreciable positive values of P_m/P_0 for small angles is to a large extent cancelled by the negative values at larger angles. A body of a

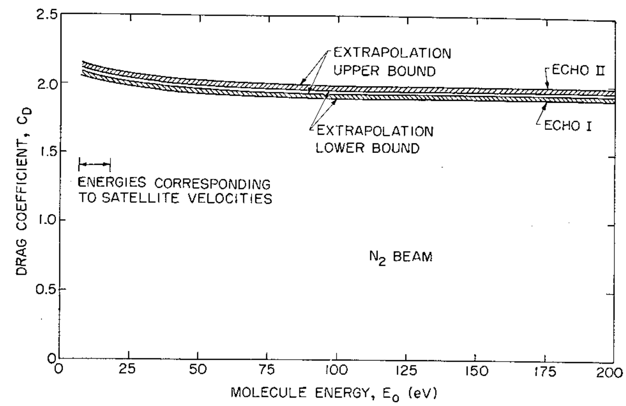

Fig. 7 Drag coefficient curves for Echo I and Echo II.

different shape might therefore give values of C_D which are significantly different from 2.

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